Finding complete solution of system of equations

Do Now: Find the null space.

$$\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix} \qquad N = \begin{bmatrix} -3 \\ 1 \end{bmatrix} a$$

$x_1 = -3$

$x_2 = 1$

$$x + 3y = 0$$
$$2x + 6y = 0$$

Find the all solutions for

$$x + 3y = 6$$
$$2\cancel{y}x + 6y = 12$$

	+3	+3	+3	+3	
x	−3	0	3	6	9
y	3	2	1	0	−1
	−1	−1	−1	−1	

$$x_p = \begin{bmatrix} 0 \\ 2 \end{bmatrix} + \quad X_N \in \left\{ \begin{bmatrix} -3 \\ 1 \end{bmatrix} a \right\}$$

$$X_c = x_p + X_n = \begin{bmatrix} 0 \\ 2 \end{bmatrix} + \begin{bmatrix} -3 \\ 1 \end{bmatrix} a$$

Any vector x_n in the nullspace can be added to a particular solution x_p. The solutions to all linear equations have this form, $x = x_p + x_n$:

Complete solution $Ax_p = b$ **and** $Ax_n = 0$ **produce** $A(x_p + x_n) = b$.

$$Ax_{1p} + BX_{2p} + CX_{3p} = b$$
$$AX_{1n} + BX_{2n} + CX_{3n} = 0$$
$$A(X_{1p} + X_{1n}) + B(X_{2p} + X_{2n}) + C(X_{3p} + X_{3n}) = b$$

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 5 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$\begin{pmatrix} 1 & 2 & 2 & 1 \\ 2 & 4 & 5 & 4 \end{pmatrix} = \begin{bmatrix} 1 & 2 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$w = 2$
$u + 2v + 2(2) = 1$
$u + 2v = 1 - 4$
$u + 2v = -3$

$$X_p = \begin{bmatrix} -5 \\ 1 \\ 2 \end{bmatrix} \qquad X_n = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

$-5, 1$
$-7, 2$
$-9, 3$

$$\begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}$$

Find the complete solution

$$\begin{aligned} y + z &= 2 \\ 2y + 2z &= 4 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

+1

y	-2	0	1	2
z	4	2	1	0

-1

$X_1 = -1$
$X_2 = 1$

$$N = \begin{bmatrix} -1 \\ 1 \end{bmatrix} a$$

$$X_c = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} a$$

/rite the complete solution

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 5 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

/rite the complete solution

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 4 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

Find the value of c that makes it possible to solve $Ax = b$, and solve it:

$$\begin{aligned} u + v + 2w &= 2 \\ 2u + 3v - w &= 5 \\ 3u + 4v + w &= c. \end{aligned}$$